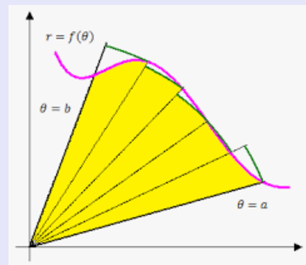
Calculus II

Lecture 26



Feb 19-8:47 AM

Class QZ 25 $\boxed{\text{open notes}}$

Find Radius, center, and interval convergence for

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \cdot n}.$$

$\boxed{\text{Make sure to check the endpoints.}}$

Ratio Test $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\cancel{(2x-1)^{n+1}}^{\,2x-1}}{\cancel{5^{n+1}}_{\,5}\cdot(n+1)}\cdot\frac{\cancel{5^n}\cdot n}{\cancel{(2x-1)^n}}\right|$

$= \lim_{n\to\infty}\left|\frac{(2x-1)\cdot n}{5(n+1)}\right| = \left|\frac{2x-1}{5}\right| \underbrace{\lim_{n\to\infty}\frac{n}{n+1}}_{\to 1} = \frac{|2x-1|}{5}$

It converges when $L < 1$ $\qquad \frac{|2x-1|}{5} < 1$

$|2x-1| < 5$

$-5 < 2x-1 < 5$

$-4 < 2x < 6$

$-2 < x < 3$

$[\ -2 \quad \cdot \quad R \quad 3\ )$

Center of conv. $\frac{1}{2}$

Radius $3 - \frac{1}{2} = \frac{5}{2}$

When $x = 3$

$$\sum_{n=1}^{\infty} \frac{(2\cdot 3-1)^n}{5^n \cdot n} = \sum_{n=1}^{\infty} \frac{5^n}{5^n \cdot n} = \sum_{n=1}^{\infty} \frac{1}{n}$$

Diverges Harmonic Series

When $x = -2$

$$\sum_{n=1}^{\infty} \frac{(2\cdot(-2)-1)^n}{5^n \cdot n} = \sum_{n=1}^{\infty} \frac{(-5)^n}{5^n \cdot n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

$(-1)^n \cdot 5^n$

Converges by A.S.T.

Interval of Convergence

$[-2, 3)$

Jul 25-7:26 AM

Binomial Series: $\longrightarrow$ K is any rational #

$$(1+x)^K = 1 + Kx + \frac{K(K-1)}{2!}x^2 + \frac{K(K-1)(K-2)}{3!}x^3 + \frac{K(K-1)(K-2)(K-3)}{4!}x^4 + \cdots$$

Converges when $|x|<1$

ex: Find binomial Series for $\sqrt[3]{1+2x}$

$$\sqrt[3]{1+2x} = (1+2x)^{1/3} \qquad |2x|<1$$

$$= 1 + \frac{1}{3}\cdot 2x + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2!}(2x)^2 + \frac{\frac{1}{3}(\frac{1}{3}-1)(\frac{1}{3}-2)}{3!}(2x)^3 + \cdots$$

$$= 1 + \frac{2}{3}x + \frac{\frac{1}{3}\cdot\frac{-2}{3}}{2}\cdot 4x^2 + \frac{\frac{1}{3}\cdot\frac{-2}{3}\cdot\frac{-5}{3}}{6}\cdot 8x^3 + \cdots$$

$$= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{\cancel{80}\,40}{27\cdot\cancel{6}\,3}x^3 + \cdots$$

$$= 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{40}{81}x^3 + \cdots \approx \sqrt[3]{1+2x}$$

If $x=1$

$$\sqrt[3]{3} \approx 1 + \frac{2}{3} - \frac{4}{9} + \frac{40}{81} - \cdots$$

$$1.442 \approx 1.716$$

Jul 25-8:26 AM

Find binomial Series for

$$\frac{1}{\sqrt{4-x}}$$

$$\frac{1}{\sqrt{4-x}} = \frac{1}{(4-x)^{1/2}} = (4-x)^{-1/2} = \left[4\left(1-\frac{x}{4}\right)\right]^{-1/2} \qquad \left|\frac{-x}{4}\right|<1$$

$$= 4^{-1/2}\cdot\left(1+\left(\frac{-x}{4}\right)\right)^{-1/2}$$

$$= \frac{1}{2}\left(1+\left(\frac{-x}{4}\right)\right)^{-1/2}$$

K, x

$$1 + Kx + \frac{K(K-1)}{2!}x^2 + \frac{K(K-1)(K-2)}{3!}x^3 + \cdots$$

$$= \frac{1}{2}\left[1 + \frac{-1}{2}\cdot\frac{-x}{4} + \frac{\frac{-1}{2}(\frac{-1}{2}-1)}{2!}\cdot\left(\frac{-x}{4}\right)^2 + \frac{\frac{-1}{2}(\frac{-1}{2}-1)(\frac{-1}{2}-2)}{3!}\cdot\left(\frac{-x}{4}\right)^3 + \cdots\right.$$

$$= \frac{1}{2}\left[1 + \frac{x}{8} + \frac{\frac{-1}{2}\cdot\frac{-3}{2}}{2}\cdot\frac{x^2}{16} + \frac{\frac{-1}{2}\cdot\frac{-3}{2}\cdot\frac{-5}{2}}{6}\cdot\frac{-x^3}{64} + \cdots\right.$$

$$= \frac{1}{2}\left[1 + \frac{x}{8} + \frac{3}{128}x^2 + \frac{15}{3072}x^3 + \cdots\right.$$

Let $x=2$

$$\frac{1}{\sqrt{4-2}} = \frac{1}{\sqrt{2}} \approx \frac{1}{2}\left[1 + \frac{2}{8} + \frac{3}{128}\cdot 2^2 + \frac{15}{3072}\cdot 2^3 + \cdots\right]$$

$$.707 \approx .691$$

Jul 25-8:36 AM

Find a binomial series for $\frac{x}{\sqrt{4+x^2}}$.

For $x=1$: $\frac{1}{\sqrt{5}}$

$$\frac{x}{\sqrt{4\left(1+\frac{x^2}{4}\right)}} = \frac{x}{2\sqrt{1+\frac{x^2}{4}}}$$

$$= \frac{x}{2}\boxed{\left(1+\underset{x}{\frac{x^2}{4}}\right)^{\overset{k}{-1/2}}}$$

$$\left|\frac{x^2}{4}\right| < 1$$

$$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots$$

$$\left(1+\frac{x^2}{4}\right)^{-1/2} = 1 + \frac{-1}{2}\cdot\frac{x^2}{4} + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)}{2}\cdot\frac{x^4}{16} + \frac{-\frac{1}{2}\left(-\frac{1}{2}-1\right)\left(-\frac{1}{2}-2\right)}{6}\cdot\frac{x^6}{64}$$

$$= 1 - \frac{1}{8}x^2 + \frac{3}{128}x^4 - \frac{15}{3072}x^6 + \cdots$$

$$\frac{x}{2}\left(1+\frac{x^2}{4}\right)^{-1/2} = \frac{x}{2} - \frac{x^3}{16} + \frac{3x^5}{256} - \frac{15}{6144}x^7 + \cdots$$

$$\frac{1}{\sqrt{5}} \approx \frac{1}{2} - \frac{1}{16} + \frac{3}{256} - \frac{15}{6144} + \cdots$$

$$.447 \approx .440$$

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